Very Important Instruction

The first 30 minutes are to be used ONLY for reading the question paper.

You MAY NOT write anything during this period, even on the Question Paper.

After 30 minutes, you will be given the answer sheets and a signal to start writing.

You will then have a further 3 hours to complete the examination.

Examination Rules

1. You have to sit at your designated desk.
2. Before the examination starts, you must check the stationery and any tools (pen, ruler, calculator) provided by the organizers.
3. You are not allowed to bring any tools except personal medicine or approved personal medical equipment.
4. You have to check the question and answer sheets provided. Raise your hand, if you find any missing sheets. Start tasks after the start whistle is blown.
5. During the examination, you are not allowed to leave the examination room except in an emergency and then you will be accompanied by a supervisor/volunteer/invigilator.
6. You are not to disturb other competitors. If you need any assistance you may raise your hand and wait for a supervisor to come to assist.
7. There will be no discussion about the examination tasks or problems. You must stay at your desk until the examination is over, even if you have finished the examination.
8. At the end of the examination time you will hear a whistle blow. You are not to write anything on the answer sheets after this stop whistle. You must leave the room quietly when asked to do so. The question and answer sheets must be left neatly on your desk.

Theory Questions

Time : 3 hrs
Marks : 30

READ THE FOLLOWING INSTRUCTIONS CAREFULLY

A. The time available is 3 hours.

B. Check that you have a complete set of the test questions and the answer sheets. The total number of questions is 5 (19 pages).

C. Write down your ID code on each page of your answer book.

D. Write your final answer in the smaller box provided. Write the steps clearly in the larger box.

Theory Questions

Time : 3 hrs
Marks : 30

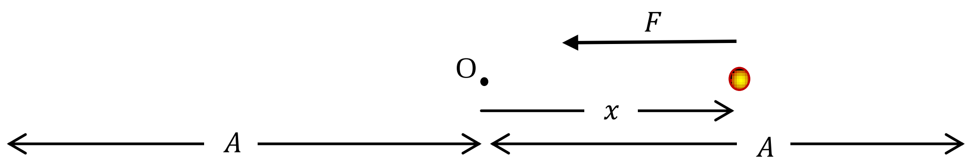
Question 1

1. Oscillations, or periodic motions, pervade our Universe. One starts from the concept of a *linear restoring force*, i.e. the force F on a body of mass m at a distance x from its equilibrium position is given by

$$F = -kx$$

where k is a *positive* constant known as the *force constant*;

The negative sign $(-)$ in the equation indicates that the force is directed towards the position O of equilibrium at $x = 0$:



Under the action of such a force, a body will execute *simple harmonic motion* (SHM), i.e. to-and-fro motion about the equilibrium position (O), with a time period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and frequency

$$\nu = \frac{1}{T}$$

The maximum displacement A of the body from its equilibrium position is called the *amplitude* of the oscillation, as shown in the figure above.

Theory Questions

Time : 3 hrs
Marks : 30

(a) Consider the Moon as a uniform solid sphere with

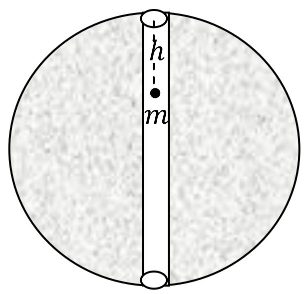
radius $R = 1.7 \times 10^6$ m,

mass $M = 7.3 \times 10^{22}$ kg, and

acceleration due to gravity at the surface $g = 1.6$ m s^{-2}.

It is known that for a spherically symmetric mass distribution, the gravitational force at a distance r from the centre is only due to the mass enclosed within a sphere of radius r with the same centre.

Now imagine the following situation. A straight narrow tunnel is dug through the Moon, passing through its centre, as indicated in the figure, and a small mass m is dropped into it from one end.



(i) The magnitude of the gravitational force experienced by the mass m at a depth h from the surface (see figure) will be

[0.5]

(A) $mg\left(1 - \frac{h}{R}\right)$ (B) $mg\left(1 + \frac{h}{R}\right)$

(C) $mg\frac{h}{R}$ (D) $mg\frac{h}{R-h}$

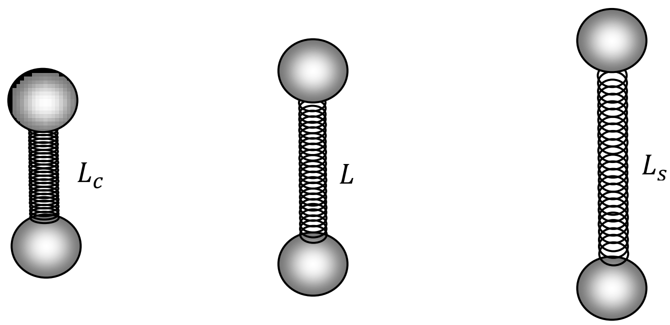
(ii) In the grid provided on your answer sheet, plot a graph of $F(r)/mg$, where $F(r)$ is the force on the mass m at a distance r from the centre of the Moon, as a function of r/R, as r varies from 0 to $2R$.

[1.0]

(iii) If $m = 0.10$ kg, what is the minimum time (in seconds) it will take, from the moment the mass m is dropped through the hole at the surface, for it to reach the centre of the Moon?

[1.0]

(b) A molecule like O_2 consists of two identical atoms held together by covalent bonding. We can think of such molecules as two identical spheres of mass m, held together by a spring that provides a linear restoring force F, with force constant k. This causes SHM of the masses along the line joining them. As a result, the molecule changes periodically from a compressed state (where the separation between the masses is minimum at L_c) to a stretched state (where the separation is maximum at L_S). In between, the force F is zero when the masses are separated by the equilibrium length L.



Obviously $L_c < L < L_s$ as the figure shows.

(i) The oxygen molecule O_2 has a force constant, $k = 1150$ N m^{-1}. The equilibrium bond length is $L = 1.5 \times 10^{-10}$ m and the change in the bond length when it is fully stretched is 6.0% of L. Calculate the vibrational energy, that is the sum of kinetic and potential energies per mole of oxygen (in kJ mol^{-1}). **[1.5]**

(Avogadro's number, $N_A = 6.023 \times 10^{23}$)

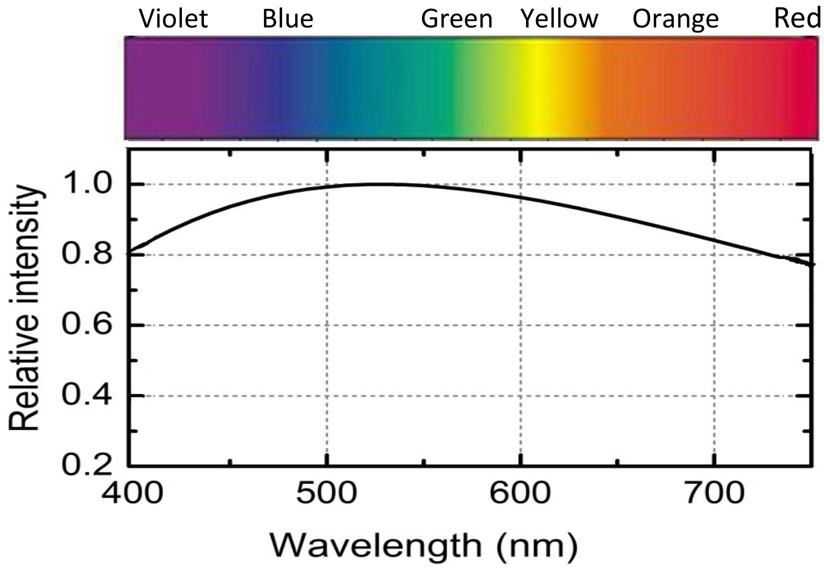
(ii) The atomic weights of the halogen elements listed in the periodic table are:

F	Cl	Br	I
19.0	35.5	79.9	126.9

Two halogen elements, X and Y, form diatomic molecules X_2 and Y_2 with force constants $k_X = 325.0$ N m^{-1} and $k_Y = 446.0$ N m^{-1} respectively. The vibration frequencies are measured to be $\nu_X = 16.7 \times 10^{12}$ Hz and $\nu_Y = 26.8 \times 10^{12}$ Hz. Identify the halogen elements X and Y by writing their symbols. Write your answer in the form X = __________ , Y = ___________ in the answer sheets. **[1.0]**

Question 2

2. Sunlight, the most important source of illumination on the Earth, contains all of the visible wavelengths, which the human eye perceives as the different colours of the spectrum. However, sunlight does not contain all wavelengths with equal intensity, as shown in the graph below. The maximum intensity is for blue-green light of wavelength about 525 nm (1 nm = 10^{-9} m).



Our perception of the colours of objects around us results mainly from the wavelength-dependent scattering or absorption of sunlight by these objects. If an object scatters/reflects back sunlight with exactly the same intensity distribution of wavelengths as above, it will appear to our eye as pure white. Any deviations from this intensity pattern in the light scattered/reflected from an object is perceived as that object having a colour.

(a) The scattering of light by particles which are much smaller than the wavelength of light, e.g. air molecules, was independently studied in the UK by Lord Rayleigh and in India by Sir C.V. Raman. They showed that if we define a *scattering efficiency* $\eta_s = I_s/I_i$, where I_i and I_s are the intensities of the incident and the scattered light respectively, then $\eta_s(\lambda) \propto \lambda^{-4}$, where λ is the wavelength of the incident light. Later, the German physicist Gustav Mie showed that if the particle sizes are comparable to the wavelength, then η_s is typically 40 times higher and is independent of the wavelength λ. Thus, one distinguishes between wavelength-dependent *Rayleigh scattering* and wavelength-independent *Mie scattering*.

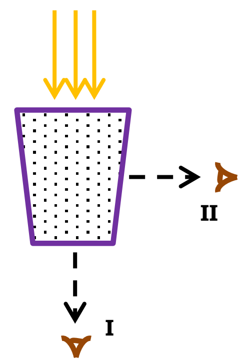
(i) If sunlight is shone through a transparent container (with walls of negligible thickness) filled with nitrogen gas, what will be the ratio of the scattered light intensity for colours corresponding to wavelengths 400 nm and 650 nm respectively? **[1.0]**

(ii) The *visibility range* R_v^0 for pure air is about 300 km for the wavelength corresponding to the blue-green colour. However, if the air is polluted by suspended materials (like smoke and dust), these scatter sunlight more efficiently than air molecules and the visibility is considerably reduced. For polluted air, the visibility range is given by

$$R_v = \frac{R_v^0}{\beta_s}$$

in terms of the *scattering loss factor* β_s, which satisfies $\beta_s \propto \eta_s C$, where C is the concentration of the scattering material and η_s is its scattering efficiency. Obviously, for pure air $\beta_s = 1$. If, after a dust storm, dust particles of size 520 nm are added to the air at 10% concentration, what will be the visibility range R_v (in km) for the blue-green colour of light? **[1.5]**

(iii) Milk is a *colloidal solution* in which droplets of liquid fat, of size around 100 nm, are suspended in water. These droplets scatter light more strongly than the water molecules, causing normal milk to appear white rather than transparent.



Consider the following experiment. A few drops of milk are added to a glass of water illuminated from above by a beam of sunlight, as shown in the figure on the right. The water turns cloudy, but some sunlight still passes through, since the concentration of milk is small. The glass is now viewed (I) from below, and (II) from the side, as shown in the figure.

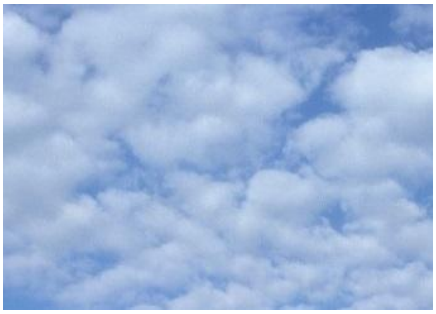
When compared to the emerging light viewed from below (I), the emerging light viewed from the side (II) will appear **[0.5]**

(A) bluish (B) orange (C) reddish (D) the same

(iv) Which of the following atmospheric phenomena is mainly governed by Mie scattering of light?

[0.5]

(A) red sunset



(B) white clouds

(C) blue sky

(D) rainbow

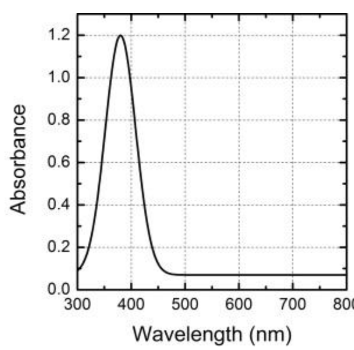
Images taken from:
(A) http://bostern.wordpress.com (B) http://www.kaneva.com
(C) http://lisathatcher.wordpress.com (D) http://www.freefoto.com

(b) Plants absorb sunlight and store the energy in chemical form, combining water and CO_2 into carbohydrate molecules by a complex process called *photosynthesis*. The discovery of photosynthesis is a long and fascinating story, starting with the Dutch physician Jan van Helmont in the 17th century. Some of the pioneering research on the physiology of photosynthesis was carried out in the 1920's by the Indian scientist, Sir J. C. Bose. A few of the details are even now under study.

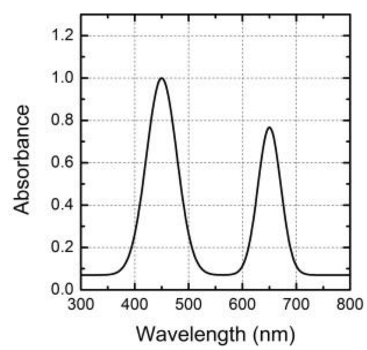
(i) The green colour of leaves and shoots of plants is usually due to the presence of chlorophyll, the compound mainly responsible for photosynthesis. Which of the following graphs depicts the correct absorbance spectrum of chlorophyll?

[1.0]

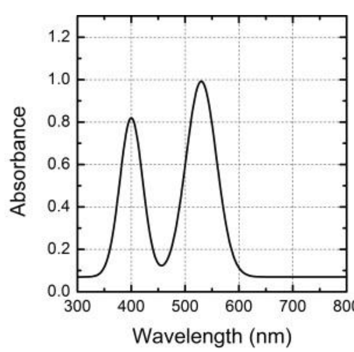
(A)



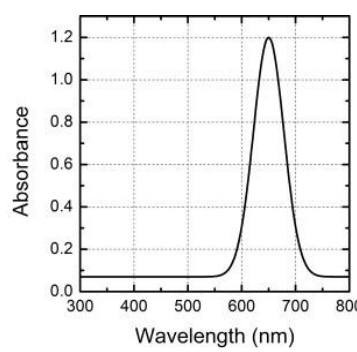
(B)



(C)



(D)



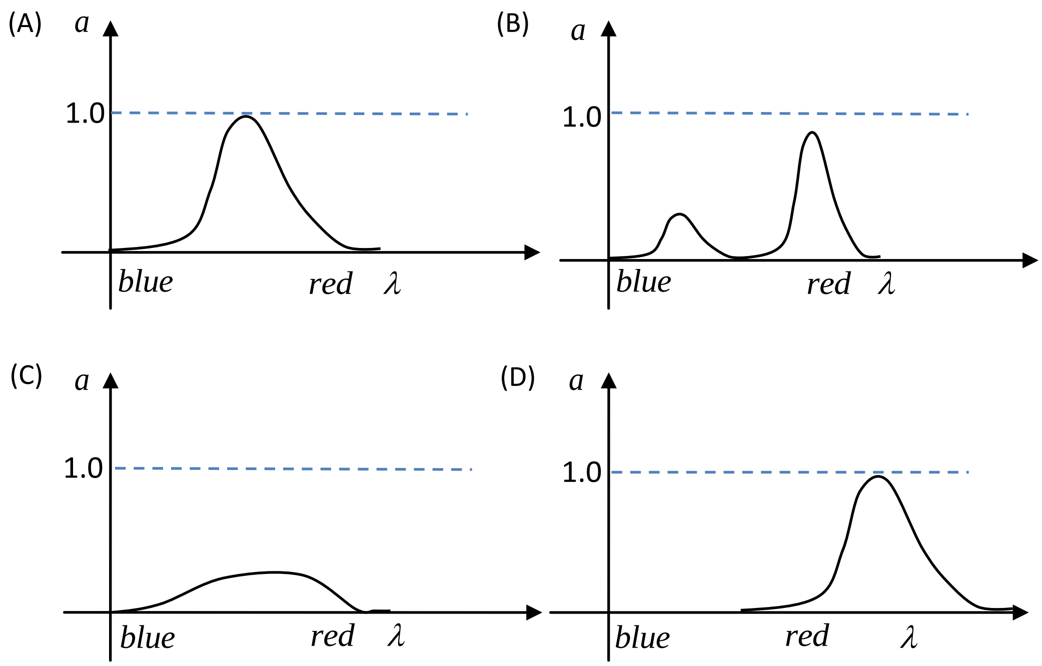
(ii) Assuming that the rate of photosynthesis is proportional to the amount of light absorbed (see above figure), what will be the wavelength (in nm) corresponding to the maximum photosynthesis rate in green plants?

[0.5]

(iii) For a long time it was believed that only plants are able to absorb solar energy and convert it into usable form. However, with the invention of solar cells, we now have artificial devices that, just like photosynthesis, also convert light into another form of energy for use in various devices.

The four graphs below represent the characteristic absorbance (a) spectra of four different materials that can potentially be used in solar cells. If these cells are designed to work in sunlight, which material will exhibit the highest conversion efficiency of sunlight into electricity?

[1.0]



Question 3

3. Maintenance of proper pH in blood and in intercellular fluids is absolutely crucial in living organisms. This is primarily because the functioning of enzymes that catalyze these processes are normally pH-dependent, and small changes in pH values may lead to serious illnesses. The pH value of human blood plasma is 7.4. The presence of CO_3^{2-}, HCO_3^- and CO_2 in body fluids helps in stabilizing pH of blood despite the addition or removal of H^+ ions by other biochemical reactions in the body.

(a) The dissociation of H_2CO_3 in blood occurs in two steps. Write down balanced equations for these two steps.

[0.5]

(b) Let the equilibrium constants for these reactions be K_1 and K_2 respectively. The values of these constants at the body temperature of $37\ ^0C$ are : $K_1 = 2.2 \times 10^{-4}$ and $K_2 = 4.8 \times 10^{-11}$.

(i) Calculate the concentration of H^+ in a solution at 37 ^{0}C, and hence its pH value, if H_2CO_3 and HCO_3^- are present in equal concentrations in mol/l in that solution.

[0.5]

(ii) Calculate the ratio of HCO_3^- and CO_3^{2-} concentrations required to maintain the pH of blood at 7.4.

[1.0]

(c) Usually in the human body, H_2CO_3 is in equilibrium with the CO_2 dissolved in the blood.

$$CO_2(\text{dissolved}) + H_2O\ (\text{liq}) \overset{K_3}{\rightleftharpoons} H_2CO_3(\text{dissolved})$$

At 37^0 C, $K_3 = 5.0 \times 10^{-3}$.
Calculate the total equilibrium constant, K' for the reaction

$$CO_2(\text{dissolved}) + H_2O\ (\text{liq}) \overset{K'}{\rightleftharpoons} HCO_3^-(aq) + H^+(aq)$$

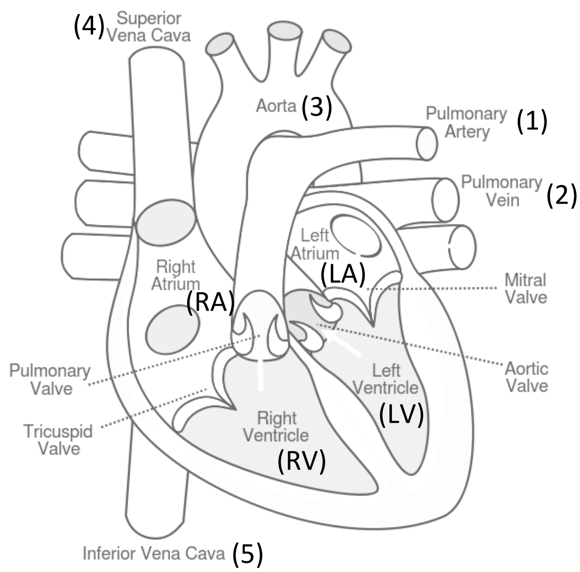
[0.5]

(d) Blood plasma contains a total carbonate buffer pool, which is a mixture of HCO_3^- and CO_2 with a total concentration of 3.4×10^{-2} M at 38 ^{0}C. At this temperature the value of the equilibrium constant K' is 1.3×10^{-6}. The concentration of the H_2CO_3 is negligible. Calculate the ratio of concentrations of CO_2(dissolved) and HCO_3^-, and their individual concentrations, in this blood sample at pH 7.4.

[1.5]

Question 4

4. The human heart has four chambers — the *left atrium*, the *right atrium*, the *left ventricle* and the *right ventricle*. These four chambers and the various blood vessels connected to the heart are shown in the diagram below.



Major blood vessels to and from the heart	Heart Chambers
1. Pulmonary artery	RA) Right Atrium
2. Pulmonary vein	RV) Right Ventricle
3. Aorta	LA) Left Atrium
4. Superior Vena Cava	LV) Left Ventricle
5. Inferior Vena Cava	

.

(a) Which of the above carry de-oxygenated blood?

[1.0]

(b) The table below shows the volume of blood V in the left ventricle of an individual at different times t during one cardiac cycle.

[0.5]

***t* (s)**	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
***V* (cm^3)**	80	89	75	60	48	47	70	80	89

What would be the individual's heart rate (in beats/minute) as calculated from the table?

(c) At different times during the cardiac cycle the various valves are open or closed so as to direct the flow of blood. Considering the data given in the above table in 4(b), what would be the correct positions of the Mitral valve and the Aortic valve at 0.2 s and 0.6 s, respectively? Fill the table provided in the answer sheet appropriately. (O = open, C=closed).

[1.5]

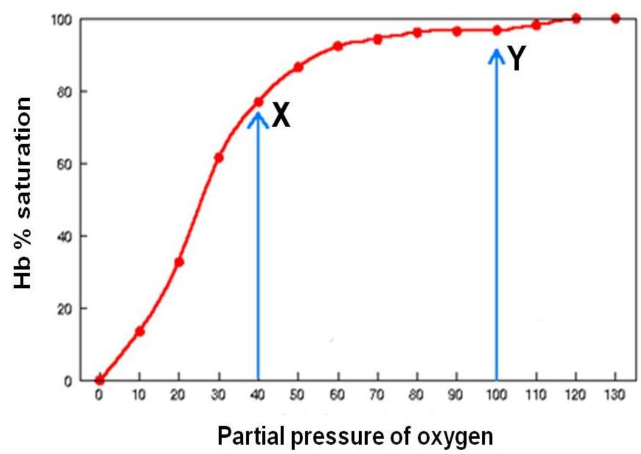
Time	Mitral valve	Aortic valve
0.2 s		
0.6 s		

(d) Blood flows from the heart into the aorta during a cardiac cycle. If the diameter of the aorta is approximately 2.4 cm, then, using the table in 4(b) calculate the average speed (in cm s^{-1}) of blood flowing into the aorta in one full cardiac cycle.

[1.0]

(e) Blood flows from the aorta and its major arteries into arterioles and fine-walled capillaries. If all the major arteries in the body have a total cross-sectional area of about 7.0 cm^2 calculate the average speed (in cm s^{-1}) in the major arteries which have the same volume of blood as the aorta flowing through them. **[0.5]**

(f) The degree of haemoglobin (Hb) saturation with oxygen may be determined by measuring the partial pressure of oxygen in various tissues of the body. The graph below shows the haemoglobin saturation level corresponding to different partial pressures of oxygen. Two such points have been marked as X and Y on the graph.



Considering the above graph, correlate the percent Hb saturation at points X and Y with approximate partial pressure of oxygen in the following areas of the body. Fill the table given in the answer sheets using X and Y as appropriate.

[1.0]

Aorta	**Renal vein**	**Alveolar space in lungs**	**Pulmonary artery**

Question 5

5. A cheetah is a wild cat, now extinct in India, but still found in some other parts of the world. Its most prominent feature is its high running speed and fast acceleration. It can accelerate from rest to its maximum running speed of about 30 m s^{-1} in just 3.0 s. (For comparison, a fast sports car like a Porsche takes about 4.0 s to attain the same speed).

Image taken from: http://www.vimeo.com

Though the cheetah can accelerate and run very fast, it cannot run a long distance at its maximum speed because it quickly gets tired. Thus, if it cannot catch its prey within that limit, it has to forgo the hunt.

(a) Consider a cheetah with mass 50 kg. It starts from rest and accelerates for 3.0 s to reach its maximum speed of 30 m s^{-1}. It then continues to run for 20 s at this speed.

(i) Calculate the average acceleration of this cheetah required to reach its maximum speed.

[0.5]

(ii) Calculate the distance travelled during the first 3.0 s, assuming that the acceleration is uniform.

[0.5]

(iii) The cheetah has to do work against friction, mostly due to air. Assume that this frictional force is always 100 N. Calculate the total mechanical work done by the cheetah during the first 23.0 s of its motion.

[1.0]

(b) During the first 23.0 s, the body temperature of the cheetah rises from 38.5 ^{0}C to 40.0 ^{0}C. Take the specific heat of the body of the cheetah to be 4.2 kJ kg^{-1} K^{-1}.

(i) If the rise in body temperature is linear during this time, calculate the total heat generated by the cheetah's metabolism. Neglect any heat loss to the surroundings.

[1.0]

(ii) Assume that some of the energy generated by the cheetah's body increases its temperature and the rest corresponds to the mechanical work done. Calculate the fraction of the total generated energy that is converted to kinetic energy.

[**1.0**]

(c) When the cheetah starts running, it generates its energy initially by aerobic respiration, where glucose is oxidised in the presence of oxygen, resulting in generation of ATP. In this process, each mole of glucose generates 36 moles of ATP, and 1130 kJ energy is released when all these ATP molecules are utilised. Running at high speeds increases the demand for oxygen, resulting in the increased breathing rate of 150 breaths per minute.

(i) Write down the balanced chemical reaction for aerobic respiration.

[1.0]

(ii) If the cheetah requires 400 kJ of energy, calculate the volume of oxygen required if all this energy is to be obtained by aerobic respiration. Take the molar volume of oxygen gas to be 24.5 litres.

[1.0]

(iii) The cheetah extracts oxygen from the air while breathing. The inhaled air (about 500 ml per breath) contains 20.0 % oxygen (by volume), while the exhaled air is assumed to contain 15.0 % oxygen (by volume). Calculate the volume of oxygen that the cheetah can use during the 23.0 s of its run, at a breathing rate of 150 breaths per minute.

[1.0]

(d) It should be clear from the answers to the above that the energy requirement of the cheetah's muscles is not met only by aerobic respiration. ATP must then be produced by anaerobic respiration, but in this only two moles of ATP are generated per mole of glucose.

(i) Anaerobic respiration converts the energy from glucose into ATP. If glucose were to be completely burnt up, one mole would release 2872 kJ of energy. What is the efficiency of anaerobic respiration compared to complete combustion of glucose?

[1.0]

(ii) If all the 400 kJ required by the cheetah for its run were to be produced by anaerobic respiration, calculate the total amount of glucose (in kg) that would be required.

[1.5]